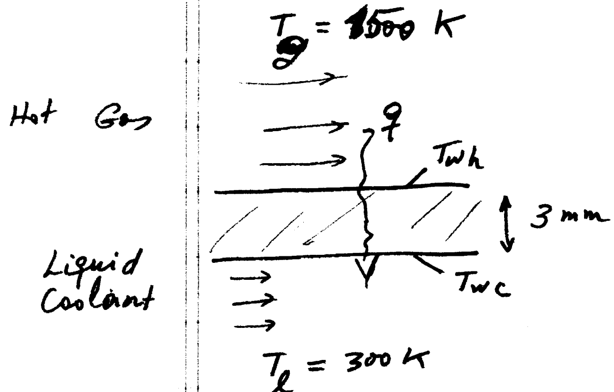
Problem 1



Consider a situation as sketched. The wall is made of steel, with a thermal conductivity $k = 40$ W/m/K. The gas is air at a pressure of 50 atm, and flows at a velocity of 700 m/s. The liquid is water, flowing at 60 m/s. The friction coefficients are estimated as $c_f = 0.0011$ on the gas side, and $c_f = 0.0033$ on the liquid side. Calculate the steady state heat flux q and the temperatures T_{wh}, T_{wc}.

Solution:

Gas side film coef.: $h_g = \rho_g u_g c_{p_g} S_{t_g}$ $\qquad S_{t_g} \simeq \frac{(c_f)_g}{2}$

$$\rho_g = \frac{P_g}{R_g T_g} = \frac{50 \times 1.013 \times 10^5}{287 \times 1500} = 11.8 \text{ kg/m}^3$$

$$c_{p_g} = \frac{\gamma}{\gamma - 1} R_g = \frac{1.4}{0.4} 287 = 1004.5 \text{ J/kg/K}$$

$$h_g = 11.8 \times 700 \times 1004.5 \times \frac{0.0011}{2} = 4560 \ \frac{W}{m^2 K}$$

Liquid side film coefficient: $h_l = \rho_l u_l c_l S_{t_l}$ $\qquad S_{t_l} \simeq \frac{(c_f)_l}{2}$

$$\rho_l = 1000 \text{ kg/m}^3$$

$$c_l = 1 \text{ Cal/g/K} = 4180 \ \frac{J}{kg K}$$

$$h_l = 1000 \times 60 \times 4180 \times \frac{0.0033}{2} = 414{,}000 \ \frac{W}{m^2 K}$$

$$q = h_g (3000 - T_{wh}) = 4560 (3000 - T_{wh})$$

$$q = 40 \frac{T_{wh} - T_{wc}}{0.003} = 13{,}300 (T_{wh} - T_{wh})$$

$$q = h_l (T_{wc} - 300) = 414{,}000 (T_{wc} - 300)$$

$$\frac{q}{414{,}000} = 3000 - T_{wh}$$

$$\boxed{q = 9.10 \times 10^{6} \ W/m^2/K}$$

and then

$$T_{wh} = 3000 - \frac{9.10 \times 10^{6}}{4560} \qquad \boxed{T_{wh} = 1004 \ K}$$

$$T_{wc} = 300 + \frac{9.10 \times 10^{6}}{414{,}000} \qquad \boxed{T_{wc} = 322 \ K}$$

Problem 2

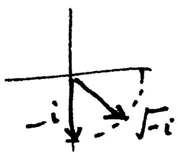
Solution

(a) Take $T = \bar{T} + Re\,[A e^{i(\omega t - ky)}]$ and substitute into $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}$. Leaving out the Re notation, we get

$$\cancel{A} i\omega \cancel{e^{i\omega(\dots)}} = \alpha \cancel{A}(-k^2)\cancel{e^{i\omega(\dots)}} \qquad k^2 = -i\frac{\omega}{\alpha}$$

$$k = \pm\sqrt{-i\frac{\omega}{\alpha}}$$

Now $\sqrt{-i} = \frac{1-i}{\sqrt{2}}$, so $k = \pm(1-i)\sqrt{\frac{\omega}{2\alpha}}$



$$e^{-iky} = e^{\mp i(1-i)\sqrt{\frac{\omega}{2\alpha}}\,y} = e^{\mp(i+1)\sqrt{\frac{\omega}{2\alpha}}\,y}$$

$$= e^{\mp\sqrt{\frac{\omega}{2\alpha}}\,y}\; e^{\mp i\sqrt{\frac{\omega}{2\alpha}}\,y}$$

The first factor is exponentially divergent in y if the lower sign is chosen, so we take the upper sign:

$$T = \bar{T} + Re\,[A\, e^{i(\omega t - \sqrt{\frac{\omega}{2\alpha}}y)}]$$

If $A = A_R + i A_I$, this is $T = \bar{T} + \Delta T\, e^{\sqrt{\frac{\omega}{2\alpha}}y}[A_R \cos(\dots) - A_I \sin(\dots)]$

Since we want $T(y=0) = \bar{T} + \Delta T \sin\omega t$, we take

$$A_R = 0 \quad ; \quad A_I = -\Delta T \; :$$

$$\boxed{T = \bar{T} + \Delta T\; e^{-\sqrt{\frac{\omega}{2\alpha}}\,y} \sin(\omega t - \sqrt{\tfrac{\omega}{2\alpha}}\, y)}$$

(b) With $\Delta T = 20\,K$, if we want the oscillation amplitude to be $1\,K$ only we need

$$e^{-\sqrt{\frac{\omega}{2\alpha}}\,y} = 0.05 \qquad y = \sqrt{\frac{2\alpha}{\omega}}\ln 20$$

We have for the soil $\alpha = \frac{k}{\rho c} = \frac{1.6}{2000 \times 2000} = 4\times10^{-7}\,m^2/s$, and

$\omega = 1.99\times10^{-7}\,s^{-1}$, so $\boxed{y = \sqrt{\frac{2\times4\times10^{-7}}{1.99\times10^{-7}}}\times \ln 20 \simeq \underline{6.0\ m}}$

Also then $\sqrt{\frac{\omega}{2\alpha}}\,y = \ln 20 \simeq 3.0$, and this is, in radians, the <u>delay</u> in reaching peak temperature. In days, $\delta t = 3.0\,\frac{365}{2\pi} = 174$ days $= 5.7$ month (peak T reached <u>in mid-winter</u>).